# Behavior of the trajectories of a single cubic operator

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In the paper for one cubic Volterra operator on a two-dimensional simplex found all the fixed points and fully understood the behavior of the trajectories generated by this operator.

One of the main tasks in the study of a dynamic system is to study the evolution of the state of the system. Usually, the "descendants" of the state of the system are determined by some law. Numerous problems of biology are solved using the theory of measure and the theory of dynamical systems. These dynamical systems are determined by iterations of nonlinear operators. We give the definition of such operators:

Let $E = \{1, 2, ..., n\}$.

Consider the set

$$S^{n-1} = \left\{ x = (x_1,\, x_2, ..., x_n) \in R^n : x_i \geq 0,\; \sum_{i=1}^{n} x_i = 1 \right\}.$$

The set $S^{n-1}$ is called the $n-1$ dimensional simplex. Each the element $x \in S^{n-1}$ is a probability measure on $E$ and its can be interpreted as a state of the biological (physical, sociological, etc.) system consisting of $n$ elements.

One of the main tasks for this system is to study the evolution of the system state. Usually, the descendants of the state of the system are determined by certain laws. For solving problems arising in mathematical genetics is used quadratic operators whose theory is currently well developed (see for example [1-3]). In [4] for one all fixed points were found on a Volterra cubic operator on a two-dimensional simplex. A description is given of the limit set of trajectories for some subclasses of such operators.

In this paper, we study dynamical systems defined by cubic operators. Fully studied trajectory of a single cubic operator on $S^2$, which arises naturally in the study of certain problems population biology.

In the simplest problem of population genetics is considered biological system $E$, consisting of $n$ species $1, 2, ..., n$. We consider that the species of parents $i, j, k$ uniquely determine the probability of each species $l$ for an immediate descendant. Denote this probability by $P_{ijk,l}$. Then $P_{ijk,l} \geq 0, \sum_{l=1}^{n} P_{ijk,l} = 1$ and the values of $P_{ijk,l}$ do not change with any permutation $i, j, k$ if the varieties are not related to gender. Population status is described by the set $x = (x_1, x_2, ..., x_n)$ probabilities of varieties. Therefore, $x \in S^{n-1}$.

## References

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